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THE IMPLIED RISK AVERSION FROM UTILITY INDIFFERENCE OPTION PRICING IN A STOCHASTIC VOLATILITY MODEL

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Abstract. In recent decades, there has been a growing interest for utility indifference based approaches to solve the question of pricing of derivatives in incomplete markets. In this paper we consider a stochastic volatility model defined as a positive non-Gaussian Ornstein-Uhlenbeck process, and price Call and Put options using the indifference methodology in the case of exponential utility. The purpose of the study is to investigate empirically the *implied risk aversion* for a representative agent in the option market, as a function of time to maturity and strike price. Our studies are based on price data for two companies, Microsoft and Volvo, where we calibrate the stochastic volatility model using historical price returns. The implied risk aversion is found by inverting numerically the indifference pricing equation, given observed option prices. The numerical inversion involves solving an integro-partial differential equation. We find that the option prices in the market are basically set by the issuer, in the sense that it is the issuer's indifference prices that matches the market prices. Since the stochastic volatility model explains the stylized facts of returns rather well, we expect the implied risk aversion to be rather flat with respect to maturity and strike price of the options. We find on the contrary a clear smile effect for short dated options, which may be explained by the issuer's fear of a market crash (in the case of the issuance of a Put option). Although the stochastic volatility model explains the heavy-tails of the returns, the crash risk seems to be unexplained by the stochastic volatility model.

1. Introduction

The volatility smile is a well-known signature for the mismatch between the theoretical Black & Scholes and the market realized price of Call options. The Black & Scholes pricing paradigm supposes a frictionless market where hedging of the option can be done continuously at no cost and (logarithmic) returns of the underlying asset are independent and normally distributed. In reality, transaction costs are incurred when trading in the market, and returns may be dependent and leptokurtic.

Many models have been suggested going beyond the geometric Brownian motion to explain the stylized facts of observed asset price returns and the volatility smile. In recent years, the stochastic volatility model of Barndorff-Nielsen and Shephard [3] has gained a lot of attention for its flexibility in explaining both the heavy-tails and the dependency

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structure of asset returns. They propose to use a geometric Brownian motion model for the asset price dynamics, where the volatility (in fact the squared volatility) follows a sum of non-Gaussian Ornstein-Uhlenbeck processes. The model is sufficiently sophisticated for a precise modeling of asset returns, besides being analytically tractable for derivatives pricing and portfolio optimization (see Benth, Karlsen and Reikvam [5] and Lindberg [14], [15]).

The crucial insight of Black & Scholes [8] and Merton [18] in their seminal papers is the independence of risk preferences in the pricing of options. However, this is strongly related to the hypothesis of completeness of the market, which in practice does not hold. A perfect hedge of an option is not possible in the real market, thus incurring a certain risk associated to issuing (or being short) an option. Therefore, the price of the option will be a reflection of the cost of a partial hedge together with a premium charged for taking on the unhedgeable risk. The latter is dependent on the issuer's risk preferences. Using the stochastic volatility model of Barndorff-Nielsen and Shephard [3] Puts us in an incomplete market, and the question of option pricing involves choosing a risk-neutral pricing measure (or an equivalent martingale measure). This can be done by appealing to techniques which takes the risk preferences of the investor directly into account.

In the last decades, utility indifference pricing has become an increasingly popular tool for a theoretical analysis of the pricing problem in incomplete markets. First proposed by Hodges and Neuberger [13] for pricing of Call options on a geometric Brownian motion stock dynamics in a market with transaction costs, it has later been used for other stock price models and different market set-ups. Closely related to our paper are Becherer [4] and Rheinländer and Steiger [21]. The utility indifference approach is usually based on the choice of an exponential utility function, since then it is in most cases possible to derive explicit prices (or at least efficiently computable prices) and these prices coincide with the Black & Scholes price when the market context "degenerates to the complete case". The exponential utility function has one parameter, measuring the risk aversion of the investor. Letting the risk aversion tend to zero we obtain a price which coincide with one induced from the minimal entropy martingale measure (see Benth and Meyer-Brandis [7] and Rheinländer and Steiger [21]). An alternative approach is to choose a martingale measure based on a structure preserving Esscher transform (see Nicolato and Venardos [19]), however, such an approach does not take into account any risk preferences of the investors explicitly (although one implicitly conjectures a risk preference by choosing this transform).

In Nicolato and Venardos [19] and Benth and Groth [6] it is demonstrated that a volatility smile is produced when using the stochastic volatility model of Barndorff-Nielsen and Shephard. In the former paper, an analysis of option prices for the S&P500 index is performed when a leverage effect is included in the dynamical model. Benth and Groth [6] price options under the minimal entropy martingale measure using a numerical solution of an integro-partial differential equation.

The purpose of this paper is to investigate the *implied risk aversion* from option prices. To the best of our knowledge, no one has so far investigated this practical approach to utility indifference pricing. Based on a hypothesis that the underlying asset price dynamics is following the Barndorff-Nielsen and Shephard model and that there is a representative agent in the market pricing options using a utility indifference method with exponential utility, we back out the implied risk aversion from theoretical prices. The theoretical prices for given risk aversion can be calculated by solving numerically a nonlinear integro-partial differential equation, being a generalization of the Black & Scholes equation. Backing out the implied risk aversion from market prices for options, we are able to study the risk aversion as a function of maturity time and exercise price of the option. Of course, if the market were using a utility indifference pricing approach, the implied risk aversion should be flat. We investigate this question empirically for options written on two stocks; Microsoft listed at NYSE and Volvo listed at the Swedish stock exchange OMX Stockholmsbörsen. The former is a very liquid asset and option, while the latter is traded in a significantly thinner market.

Using historical time series for the asset prices and trading volumes, we fit the stochastic volatility model. The estimation procedure is based on a technique developed by Lindberg [16], efficiently calibrating the stochastic volatility model with a high degree of statistical precision. From this we calculate option prices by solving an integro-partial differential equation using advanced numerical methods. Our results indicate that prices are in favour of the issuer, since the traded price observations are above the minimal entropy martingale measure prices. We find also a smile, or rather a smirk, effect in the implied risk aversion. The result tells us that even when using a highly sophisticated stochastic volatility model which explains the dependency and distributional properties of the returns close to perfect together with a pricing approach taking risk preferences into account, there are still risks unaccounted for. The obvious explanation is of course that we have not taken transaction costs into account. However, this can not be the only reason, since a large part of the option trades are naked, that is, the short position is not covered by a hedge, thus making transaction costs irrelevant. There is also an interesting shape of the implied risk aversion which may be explained by differences in out-of and in-the money positions. We find that although our stochastic model for the asset prices includes heavy-tailed returns, the market is pricing in a premium for potential crashes.

The paper is organized as follows. We introduce the Barndorff-Nielsen and Shephard stochastic volatility model in Section 2 together with short sections about the minimal entropy martingale measure and utility indifference pricing. Section 3 contains the estimation of the parameters in the model. We solve the indifference pricing problem numerically in Section 4, *i.e.* calculate theoretical option prices. Finally in Section 5 we use the numerical framework and market prices to backtrack the implied risk aversion in the market for the two option classes studied.

2. The model

2.1. **Model definitions.** For $0 \leq t \leq T < \infty$, we assume as given a complete probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\{\mathcal{F}_t\}_{0 \leq t \leq T}$ satisfying the usual conditions. We take a subordinator L, and denote its Lévy measure by $l(\mathrm{d}z)$. A subordinator is defined to be a Lévy process taking values in $[0, \infty)$, which gives that its sample paths are increasing.

The Lévy measure l of a subordinator satisfies the condition

$$\int_{0+}^{\infty} \min(1,z) l(\mathrm{d}z) < \infty.$$

We assume that we use the cádlág version of L.

Denote by Y the OU stochastic process whose dynamics are governed by

$$\mathrm{d}Y(t) = -\lambda Y(t)\,\mathrm{d}t + \mathrm{d}L(\lambda t), \tag{1}$$

where $\lambda > 0$ denotes the *rate of decay.* We call processes with these dynamics *news processes.* The unusual timing of L is chosen so that the marginal distribution of Y will be unchanged for any value of λ.

The stationary news process Y can be written as

$$Y(t) = Y_0 e^{-\lambda t} + \textstyle\int_0^t e^{-\lambda(t-u)}\,\mathrm{d}L(\lambda u), \quad t \geq 0, \tag{2}$$

where $Y_0 := Y(0)$. The variable Y_0 has the stationary marginal distribution of the process and is independent of $L(t) - L(0)$, $t \geq 0$. Further, if $Y_0 \geq 0$, then $Y(t) > 0\ \forall t \in [0,T]$, since L is non-decreasing. We set $L(0) = 0$. In general, the Y can be expressed as linear combinations of the Ornstein-Uhlenbeck processes in Equation (2). However, we consider for simplicity the case with only one such process. The square root of the process Y is called the volatility, denoted by $\sqrt{Y}$.

Consider a Wiener process W independent of L. We use the filtration

$$\{\mathcal{F}_t\}_{0\leq t\leq T} := \{\sigma\left(W(t), L(\lambda t)\right)\}_{0\leq t\leq T},$$

to make the OU process and the Wiener process simultaneously adapted. Define the stock price S to have the dynamics

$$\mathrm{d}S(t) = S(t)\left(\left(\mu + \beta Y(t)\right)\,\mathrm{d}t + \sqrt{Y(t)}\,\mathrm{d}W(t)\right),$$

where μ is the *constant mean rate of return*, and β is the *skewness* parameter. This dynamics implies the explicit stock price process

$$S(t) = S(0)\exp\left(\int_0^t \left(\mu + \left(\beta - \tfrac{1}{2}\right)Y(u)\right)\,\mathrm{d}u + \int_0^t \sqrt{Y\left(u\right)}\,\mathrm{d}W(u)\right). \tag{3}$$

The model allows for the increments of the *logreturns* $R\left(t\right) := \log\left(S(t)/S(0)\right)$, to have semi-heavy tails as well as both volatility clustering and skewness. The increments of the logreturns R are stationary since

$$R\left(s\right) - R\left(t\right) = \log\left(\frac{S\left(s\right)}{S\left(0\right)}\right) - \log\left(\frac{S\left(t\right)}{S\left(0\right)}\right) = \log\left(\frac{S\left(s\right)}{S\left(t\right)}\right) \stackrel{\mathcal{L}}{=} R\left(s-t\right), \tag{4}$$

where " $\stackrel{\mathcal{L}}{=}$ " denotes equality in law.

We assume the usual risk-free bond dynamics

$$\mathrm{d}B(t) = rB(t)\,\mathrm{d}t,$$

with a constant interest rate $r > 0$.

2.2. **The minimal entropy martingale measure.** We recall a few results from [7] for the convenience of the reader.

Assume that the Lévy measure l satisfies

$$\int_1^\infty \{e^{\alpha z}-1\}\, l(\mathrm{d}z) < \infty,$$

for the constant

$$\alpha = \frac{\beta^2}{\lambda}\left(1-e^{-\lambda T}\right).$$

It is shown in [7] that under this condition on l, the density process of the minimal entropy martingale measure (MEMM), denoted by $\mathbb{Q}_{ME}$, can be represented as

$$Z(t) := Z^W(t)Z^L(t),$$

where

$$Z^W(t) = \exp\left(-\int_0^t \frac{\mu+\beta Y(u)}{\sqrt{Y(u)}}\,\mathrm{d}W(u) - \int_0^t \frac{1}{2}\frac{(\mu+\beta Y(u))^2}{Y(u)}\,\mathrm{d}u\right),$$

and

$$Z^L(t) = \exp\left(\int_0^t\int_0^\infty \log\delta\left(Y(u),z,u\right)N(\mathrm{d}z,\mathrm{d}u) + \int_0^t\int_0^\infty \left(1-\delta\left(Y(u),z,u\right)\right)l(\mathrm{d}z)\mathrm{d}u\right),$$

for the Poisson random measure $N(\mathrm{d}z,\mathrm{d}u)$ of L. The function $\delta(y,z,t)$ is defined as

$$\delta(y,z,t) := \frac{Ht(t,y+z)}{H(t,y)},$$

where

$$H(t,y) = \mathbb{E}\left[\exp\left(-\frac{1}{2}\int_t^T\left\{\frac{\mu^2}{Y(u)}+2\mu\beta+\beta^2Y(u)\right\}\mathrm{d}u\right)\Big|\, Y(t)=y\right], \tag{5}$$

for $(t,y)\in[0,T]\times\mathbb{R}_+$. It turns out that $H(t,y)$ solves the integro-pde

$$\partial_t H(t,y) - \frac{1}{2}\left(\frac{\mu^2}{y}+2\mu\beta+\beta^2 y\right)H(t,y) + \mathcal{L}_\sigma H(t,y) = 0, \tag{6}$$

for $(t,y)\in[0,T)\times\mathbb{R}_+$ with

$$\mathcal{L}_\sigma H(t,y) = -\lambda y\partial_y H(t,y) + \lambda\int_{0+}^\infty \{H(t,y+z)-H(t,y)\}\, l(\mathrm{d}z),$$

and terminal data $H(T,y)=1$, $y\in\mathbb{R}_+$.

2.3. **Utility indifference pricing.** The concept of utility indifference pricing was proposed by [13]. The idea springs from realizing that in incomplete markets, arbitrage pricing theory does not give unique option prices, so additional criteria are required. The utility indifference price for an issuer of an option is the price for which she is indifferent between selling a contract or entering the market by her own account. The approach requires that the investor chooses a utility function, the most common one being the exponential utility function

$$U(x) = 1 - \exp\left(-\gamma x\right),$$

where $\gamma > 0$ is the risk aversion parameter. This choice has the advantage that the price of the option becomes independent of the issuer's wealth, but most of all it allows for explicit computations. For a mathematical foundation for the following analysis, we refer to Becherer [4], Benth and Meyer-Brandis [7] and Rheinländer and Steiger [21].

We denote by $\mathcal{A}$ the set of $\mathcal{F}_t$-adapted controls π for which there exists a wealth process X_t^π that solves

$$\mathrm{d}X(u) = X(u)\left[\pi\left(u\right)\left(\mu + \beta Y(u)\right)\,\mathrm{d}u + r\,\mathrm{d}u + \pi\left(u\right)\sqrt{Y(u)}\,\mathrm{d}B(u)\right], \quad X(t) = x.$$

The value function for the optimal control problem, if the investor does not issue a claim, is

$$V^0(t,x,y) = \sup_{\pi\in\mathcal{A}} \mathbb{E}\left[1 - \exp\left(-\gamma X(T)\right) \middle| X(t) = x, Y(t) = y\right].$$

If the investor issues a claim $f(S(T))$, the value function becomes

$$V(t,x,y,s) = \sup_{\pi\in\mathcal{A}} \mathbb{E}\left[1 - \exp\left(-\gamma\left(X(T) - f(X(T))\right)\right) \middle| X(t) = x, Y(t) = y, S(t) = s\right].$$

Hence the utility indifference price for the claim $f(S(T))$ is given by the unique solution $\Lambda(t,y,s)$ to the equation

$$V^0(t,x,y) = V(t, x + \Lambda^{(\gamma)}(t,y,s), y, s).$$

Provided the value functions are sufficiently smooth we can apply the dynamic programming method to solve the two stochastic control problems. In the process we derive the Hamilton-Jacobi-Bellman (HJB) equations associated with the value functions. It happens that equation (6) corresponds to the first case, when no claim is issued.

Solving the second value function, when a claim is issued, we arrive at the HJB-equation for the utility indifference price of the option. The form of the integro-pde depends on whether we look at the problem from the seller or the buyers side, differing only in sign of terms in the equation. The integro-pde for the price $\Lambda^{(\gamma)}$ for the issuer of a claim becomes

$$\begin{aligned} r\Lambda^{(\gamma)} &= \Lambda_t^{(\gamma)} + \frac{1}{2}ys^2\Lambda_{ss}^{(\gamma)} - \lambda y\Lambda_y^{(\gamma)} + rs\Lambda_s^{(\gamma)} \\ &\quad + \lambda\int_0^\infty \frac{1}{\gamma}\left\{\exp\left(\gamma\left(\Lambda^{(\gamma)}(t,y+z,s) - \Lambda^{(\gamma)}(t,y,s)\right)\right) - 1\right\}\frac{H(t,y+z)}{H(t,y)}l(\mathrm{d}z), \end{aligned} \tag{7}$$

with $\Lambda^{(\gamma)}(T,y,s) = f(s)$, for $(t,y,s) \in [0,T) \times \mathbb{R}_+^2$, where H is given by Equation (6). Hence, to obtain the prices $\Lambda^{(\gamma)}$ one has to solve a system of two coupled integro-pde. For completeness, we also include the integro-pde for the indifference price of the buyer of the option, denoted $\widehat{\Lambda}^{(\gamma)}$:

$$\begin{aligned} r\widehat{\Lambda}^{(\gamma)} &= \widehat{\Lambda}_t^{(\gamma)} + \frac{1}{2}ys^2\widehat{\Lambda}_{ss}^{(\gamma)} - \lambda y\widehat{\Lambda}_y^{(\gamma)} + rs\widehat{\Lambda}_s^{(\gamma)} \\ &\quad - \lambda\int_0^\infty \frac{1}{\gamma}\left\{\exp\left(-\gamma\left(\widehat{\Lambda}^{(\gamma)}(t,y+z,s) - \widehat{\Lambda}^{(\gamma)}(t,y,s)\right)\right) - 1\right\}\frac{H(t,y+z)}{H(t,y)}l(\mathrm{d}z), \end{aligned}$$

with $\widehat{\Lambda}^{(\gamma)}(T,y,s)=f(s)$, for $(t,y,s)\in[0,T)\times\mathbb{R}_+^2$.

The lowest acceptable utility indifference price for an issuer of a claim is reached when the risk aversion γ tends to zero. This price coincides with the arbitrage free price under MEMM, but also with the maximal utility indifference price for a buyer of the same claim. This makes MEMM particularly interesting to study. In the risk aversion limit $\gamma\downarrow 0$, equation (7) simplifies to (see [7])

$$\begin{aligned} r\Lambda &= \Lambda_t+\frac{1}{2}ys^2\Lambda_{ss}-\lambda y\Lambda_y+rs\Lambda_s \\ &+\lambda\int_0^\infty \gamma\left(\Lambda(t,y+z,s)-\Lambda(t,y,s)\right)\frac{H(t,y+z)}{H(t,y)}l(\mathrm{d}z). \end{aligned}\tag{8}$$

We have used the short-hand notation $\Lambda^{(0)}:=\Lambda$ here.

It is well known that a higher risk aversion leads to higher prices, so if the option prices we observe in the market is higher than the prices under MEMM, we can assume the buyer has a risk aversion $\gamma>0$. If they, on the other hand, are lower the same applies but for the seller. Using Equation (7), market prices and a root finding algorithm, we shall find the *implied risk aversion* from the market.

3. On estimating the BNS model to price and volume data

In this section we use the approach from [16] to analyze observed asset prices from two stocks Microsoft and Volvo. The estimation approach of [16] involves using both observed stock prices as well as the traded volume of the asset. The latter is used to get information for the volatility variations.

We have available time series of daily adjusted closing prices and daily trading volume for the Microsoft stock traded at the New York Stock Exchange in the period 1 January 2004 to 18 September 2006. For the Swedish company Volvo we also have daily adjusted closing prices and daily traded volume of its B shares collected from the OMX Stockholmsbörsen over the time period 1 August 2004 to 30 December 2005.

We start with presenting a discrete time version of the BNS model together with the method to fit this to the observations. Next, we apply the estimation method for the available data sets.

Assume that the logreturns

$$R^c(\Delta),R^c(2\Delta)-R^c(\Delta),...,R^c(d\Delta)-R^c((d-1)\Delta),$$

are observed, with R^c defined by Equation (4). From now on, Δ is assumed to be one day, and the number of consecutive observations in our time series data is $d+1$.

It is reasonable to assume that the approximation

$$\int_{t-\Delta}^t\sqrt{Y(s)}\,\mathrm{d}W(s)\approx\sqrt{Y(t)}\,\varepsilon,\tag{9}$$

holds, with $\varepsilon(t)\sim N(0,1)$ being *i.i.d.*, unless some λ_j are large so that the volatility processes will be volatile. The model in Equations (3) and (4) then take the discrete time form

$$R(t) = \mu + \beta Y(t) + \sqrt{Y(t)}\,\varepsilon(t), \tag{10}$$

where $t = 1, 2, ...,$ and $\varepsilon(\cdot)$ is a sequence of independent $N(0, 1)$ variables.

It was argued in [16] that one should not try to fit the logreturns directly to data. This is due to the severe parameter instability, or large flexibility, of many of the marginal distributions typically used in finance, such as the Normalized Inverse Gaussian (NIG) distribution. Instead, it was proposed that one should try to measure Y with parameters μ and β such that the *empirical normalized logreturns*

$$\xi(\cdot) := \frac{R(\cdot) - (\mu + \beta Y(\cdot))}{\sqrt{Y(\cdot)}} \tag{11}$$

are *i.i.d.* and $N(0, 1)$. If we can do this, it is easy to model Y within the framework in [3], thanks to the large flexibility of the BNS model. This approach verifies the validity of the discrete time model, and allows us to understand better the structure of the process that generated the returns $R(\cdot)$. It is important to get $\xi(\cdot)$ and the model for $Y(\cdot)$ correct, since it is these quantities that generate the model, and hence contain the key to the understanding of it. The next priority is to get the parameters for the distribution of $Y(\cdot)$. Equation (10) gives then an *implied* distribution of the returns $R(\cdot)$ that we have a good comprehension of. The procedure is illustrated by using $NIG(\mu, \beta, \delta, \gamma)$ as the marginal distribution of the returns $R(\cdot)$. This implies that the that the volatility $Y(\cdot)$ has an Inverse Gaussian distribution $IG(\delta, \gamma)$. We proceed as follows.

1. Find volatility processes $Y(\cdot)$ and parameters μ and β for each stock so that the normalized returns $\xi(\cdot)$ become independent $N(0, 1)$. For this purpose, we assume that the discrete time volatility processes $Y(\cdot)$ is a constant times some measure of trading intensity $z(\cdot)$ on each trading day, *i.e.*,

$$Y(\cdot) = \theta z(\cdot). \tag{12}$$

The idea of this model is to try to verify that a function of some measure of trading intensity can by used as $Y(\cdot)$ in Equation (11) to obtain $\xi(\cdot)$ that are *i.i.d.* and $N(0, 1)$. We model then $Y(\cdot)$ within the framework in [3]. If we can do this, we have asserted that our continuous time stochastic volatility model is reasonable. Further, we get an economical interpretation of the volatility.

Note that we do not claim that the number of trades, the number of traded stocks, or any other measure of trading intensity, can always be used to model the volatility for all stocks. However, we have experienced that very often one can use such measures to obtain good estimates of the volatility for relatively long periods of time. Advantages are that we can get stable parameter estimates easily and with only daily data.

The next step of the estimation procedure is:

2. Estimate parameters δ and γ so that the empirical distributions of $Y(\cdot)$ from Equation (12) fit the $IG(\delta, \gamma)$ distribution.

Hence, we have specified the NIG-distribution for $R(\cdot)$. We could do this estimation simultaneously for IG and NIG. However, since the NIG-distribution is very flexible and unstable, we know that even if we would get a slightly better fit this way, it would be at the cost of less understanding of the process.

The third an final step in the calibration of the BNS-model says:

3. Use the estimates of the volatility processes $Y(\cdot)$ to estimate the rates of decay λ_j. This is done by matching the empirical autocorrelation function with the of autocorrelation function of the continuous time volatility process Y.

The autocorrelation $\rho_{\sqrt{Y}}$ of the volatility process becomes

$$\rho_{\sqrt{Y}}(h) = \frac{Cov(Y(h),Y(0))}{Var(Y(0))} = \exp(\lambda |h|), \quad h \geq 0.$$

The rate of decay λ is therefore obtained from the discrete time volatilities $\sqrt{Y}(1), ..., \sqrt{Y}(d)$, by minimizing the least squared distance between the theoretical and empirical autocorrelation functions.

We now move on to implement this statistical approach to calibrating the stock price process and its stochastic volatility model to observed price and volume data. We discuss mainly the statistical analysis for the Microsoft stock, and report only some major results for the Volvo stock.

3.1. **Microsoft.** For Microsoft, we choose

$$Y = \theta \times (\text{Normalized Traded Volume})^{3/2} \ ,$$

as a simple model for the volatility, where the exponent 3/2 was picked *ad hoc* since it gave nice normalized returns. This parameter could of course also be made part of the optimization algorithm, but in our experience, the results remain approximately the same for exponents between 1 and 2. Further, we have no economical intuition as to why we should prefer one exponent over another. To get a better scaling, we use 'Normalized Traded Volume' which is the traded volume divided by its standard deviation. This model turns out to give a good fit. Judging from Figures 1 and 2, we have little reason to suspect that ξ would not come from an *i.i.d.* sample, although the autocorrelation for $|\xi(\cdot)|$ shows a significant positive dependence on a few too many lags. Moreover, the empirical cdf of ξ, and the normal probability plot in Figure 3, indicates a very nice fit of ξ to the normal distribution. In particular compared to the normal probability plot of the raw returns, see Figure 3. Further, ξ pass the Kolmogorov-Smirnov test for normality with a p-value of 0.97, as well as the Jarque-Bera normality test based on skewness and kurtosis with a p-value of 0.12. Since the mean value parameters μ and β are connected through the relation

$$\mathbb{E}[R] = \mu + \beta \mathbb{E}[Y] \ , \tag{13}$$

it is misleading to look at confidence intervals for these parameters. Instead, we check robustness of the results by testing the hypothesis $H_0 : \mu = \beta = 0$. Under this hypothesis, the Kolmogorov-Smirnov and the Jarque-Bera tests give the p-values 0.83 and 0.09 respectively, which indicates that the model is not very sensitive to these parameters. Under H_0, we can use standard normal statistical theory to get a 95% confidence interval for θ. The interval is $[2.51 \times 10^{-5}, 3.12 \times 10^{-5}]$. Since the effect of μ and β is small, the confidence

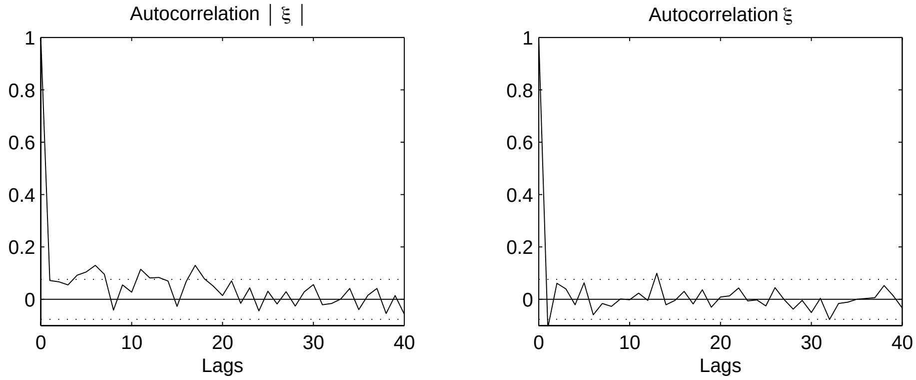


FIGURE 1. *Left:* The estimated autocorrelation function for the absolute normalized returns $|\xi|$ for the Microsoft stock from January 1, 2004, to September 18, 2006. *Right:* The estimated autocorrelation function for the normalized returns for Microsoft during the same time period. The figures show the first 40 lags, and the straight lines parallel to the x-axes are the asymptotic 95% confidence bands $\pm 1.96/\sqrt{number\ of\ observations}$.

interval for $\hat{\theta}$ without the hypothesis H_0 will be similar. However, it is hard to calculate this exactly.

The implied NIG-distribution and the estimated IG-distribution fit their empirical densities well, see Figure 4. In addition, the volatility process Y has the characteristic look of a news process, see Figure 5. The parameter estimates are $\hat{\mu}_M = -7.70 \times 10^{-4}$, $\hat{\beta}_M = 8.65$, $\hat{\delta}_M = 0.0186$, $\hat{\gamma}_M = 194$, $\hat{\lambda}_M = 1.14$, and $\hat{\theta}_M = 2.78 \times 10^{-5}$.

3.2. **Volvo.** For Volvo, the model $Y = \theta \times (\text{Normalized Traded Volume})^2$ was used, where, analogous to above, the 2 in the exponent was chosen because it gave good normalized returns, but could equally well have been part of the optimization procedure. The same figures as in the analysis of the Microsoft stock all looked good, see for example Figure 6. The p-values for the Kolmogorov-Smirnov test and the Jarque-Bera test were 0.73 and 0.72, respectively. The parameter estimates are $\hat{\mu}_V = 6.21 \times 10^{-4}$, $\hat{\beta}_V = 1.27$, $\hat{\delta}_V = 0.0116$, $\hat{\gamma}_V = 54.2$, $\hat{\lambda}_V = 0.83$, and $\hat{\theta}_V = 6.63 \times 10^{-5}$.

4. SOLVING THE INTEGRO-PDE FOR INDIFFERENCE PRICING NUMERICALLY

We saw in Section 2.3 that the utility indifference price of a claim could be represented as the solution of a coupled system of integro-pdes. Numerical solution of integro-pdes in the context of finance has been studied extensively over the last decade. For Lévy processes the finite difference method has been used by Andersen and Andreasen [1] and Cont and Voltchkova [9]. Finite element methods for Lévy driven processes was studied by Matache, Petersdorff and Schwab [17] and stochastic volatility models driven by Brownian motions by Hilber, Matache and Schwab [12]. For the BNS model we build upon work by Benth and Groth [6], who use finite differences to solve Equation (8) and find option prices under

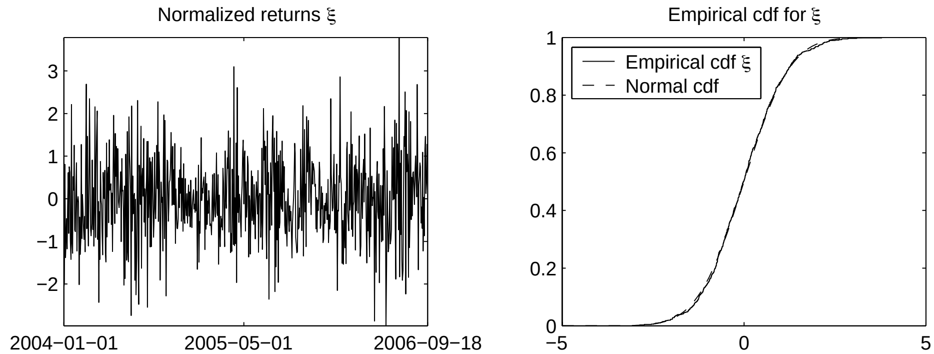


FIGURE 2. *Left:* The normalized returns ξ for the Microsoft stock during January 1, 2004, to September 18, 2006. *Right:* The empirical cdf for ξ for Microsoft during the same time period, and the standard normal cdf.

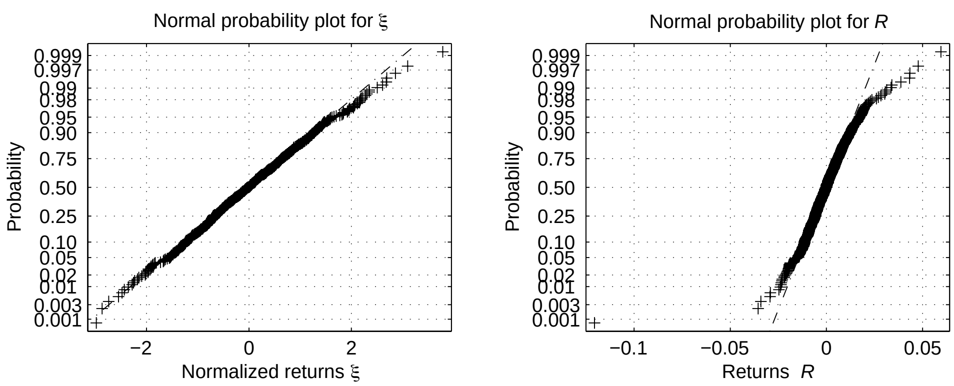


FIGURE 3. *Left:* The normal probability plot of the normalized returns ξ for the Microsoft stock during January 1, 2004, to September 18, 2006. *Right:* The normal probability plot of the returns for Microsoft during the same time period. The theoretical quantiles are on the y-axes.

the minimal entropy martingale measure. Since we are interested in both the MEMM prices and those derived from general risk aversion in equation (7), we must adapt the methodology used in [6].

Solving Equation (8) with finite differences implies restricting the equation to a finite grid. The problem is in its nature unbounded, since the stock price and volatility in theory could have arbitrary large value. Because of the restriction to a finite grid we need to find appropriate boundary conditions where necessary. We also have to approximate the non-local integral term on a sufficient range of points. The approximation should be able to capture the main influence from the integral since the Lévy measure will kill off the integral for sufficient large z. For simplicity we use a simple trapezoid scheme. To handle the two-dimensional problem we use Gudonov operator splitting [11] following suggestions

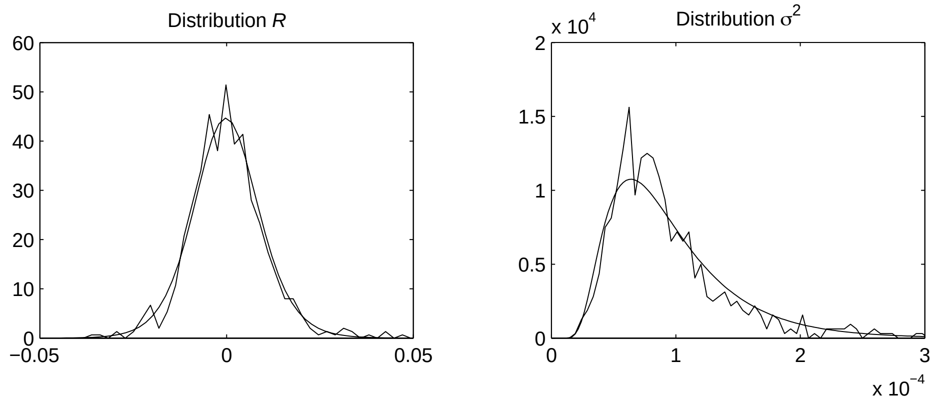


FIGURE 4. *Left:* Plot of empirical density of the returns and the implied NIG density obtained from the estimated IG density for the Microsoft stock during January 1, 2004, to September 18, 2006. *Right:* Plot of empirical density of $\hat{\theta} *$ (Number of trades per day) and the estimated IG-density during the same time period.

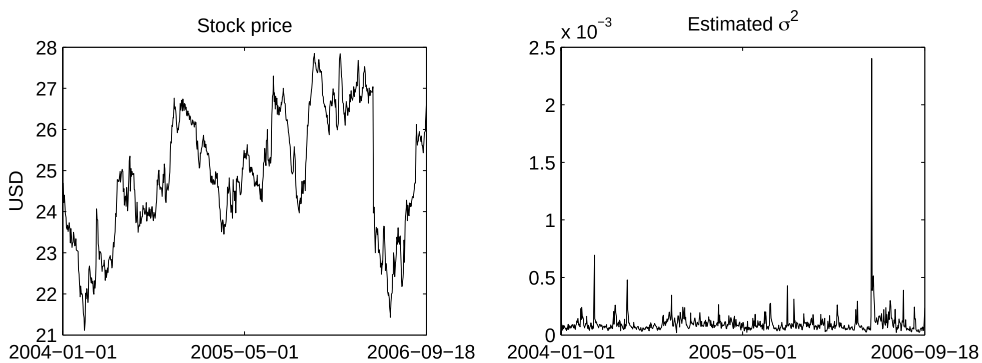


FIGURE 5. *Left:* The price process in USD for the Microsoft stock from January 1, 2004, to September 18, 2006. *Right:* The estimated volatility process $\hat{\theta} *$ (Number of trades per day)$^{\frac{3}{2}}$ for Microsoft during the same time period.

by Strang [22]. This gives us two one-dimensional equations which we solve iteratively. It is possible that the subordinator $L(t)$ is of infinity activity, which gives the Lévy measure a singularity at zero. Since this can not be handled by the trapezoid scheme we add a diffusion term to make up for the part of the integral close to zero.

Regarding the general risk aversion equation, for numerical stability we make the change of variable

$$\Lambda^{(\gamma)}(t, y, s) = \frac{1}{\gamma} \ln h^{(\gamma)}(t, y, s).$$

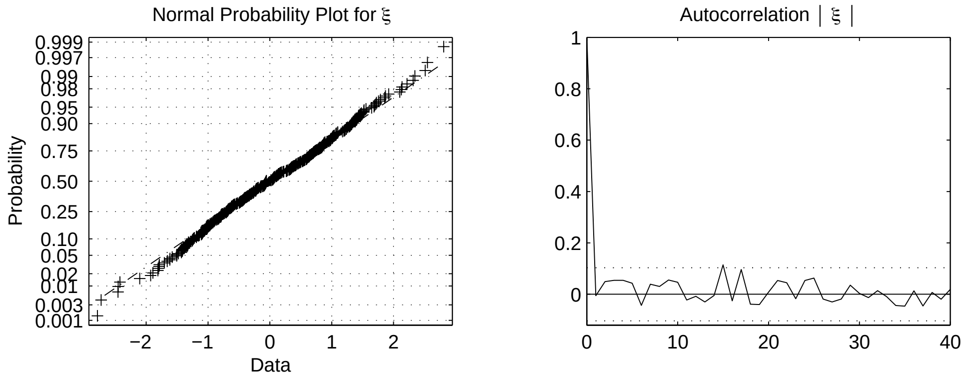


FIGURE 6. *Left:* The normal probability plot of the normalized returns ξ for Volvo B during August 1, 2004, to December 30, 2005. The theoretical quantiles are on the y-axes. *Right:* The estimated autocorrelation function for the absolute normalized returns $|\xi|$ for the Volvo B stock from August 1, 2004, to December 30, 2005. The figure shows the first 40 lags, and the straight lines parallel to the x-axes are the asymptotic 95% confidence bands $\pm 1.96/\sqrt{number\ of\ observations}$.

This transforms Equation (7) into the non-linear integro-pde

$$\partial_t h^{(\gamma)} + \frac{1}{2} y s^2 \partial_{ss} h^{(\gamma)} + r s h^{(\gamma)} - \frac{1}{2} y s^2 \frac{(\partial_s h^{(\gamma)})^2}{h^{(\gamma)}} + \mathcal{L}_Y^{memm} h^{(\gamma)} = r h^{(\gamma)} \tag{14}$$

with initial condition $h^{(\gamma)}(T, y, s) = \exp(\gamma f(s))$. Here

$$\mathcal{L}^{memm} h(t,y) = -\lambda y h(t,y) + \lambda \int_0^\infty \{h(t, y+z) - h(t,y)\} \frac{H(t, y+z)}{H(t,y)} \ell(\mathrm{d}z)$$

This is a nonlinear integro-pde, where the only nonlinearity is in the quadratic term $(\partial_s h^{(\gamma)})^2 / h^{(\gamma)}$. We remark that this non-linear term is less severe to handle than the appearance of an exponential term in the integrand. For Equation (8) we use implicit schemes, deriving a Lax-Wendroff scheme for the non-homogeneous equation involving the integral. For Equation (14) we need to use an explicit scheme for the non-linear one-dimensional equation. This forces us to take significantly shorter time steps when running the solver.

Benth and Groth [6] derive suitable boundary condition for the integro-pde, which we have collected in Table 1. The Dirichlet boundary conditions mean using Black-Scholes prices at the boundaries, *i.e.* as the variables goes to infinity the prices will adjust to the corresponding Black-Scholes prices. Further motivation for the choice of boundary conditions and the methodology applied to handle the integral can be found in [6]. Boundary conditions for Equation (14) are similar.

For the sake of visualization we have used interpolation between the points in the data set where necessary to plot the result.

Boundary	Boundary condition
$s = 0$	Dirichlet
$s = S_{max}$	Dirichlet
$y = 0$	von Neumann
$y = Y_{max}$	Dirichlet

Table 1. Boundary conditions for the integro-pdes. The Dirichlet condition is to use appropriate Black-Scholes prices while we have a strong reflection giving a von Neumann condition at $y = 0$.

4.1. **MEMM prices.** Given the parameters estimated above we can use the implemented solver to calculate option prices under MEMM. We know that theoretically the MEMM price is the highest price the buyer and lowest price the seller can agree on. Comparing with bid/ask-prices gives us a pointer whether the market is in favour of either one of them. If the MEMM prices if below the bid prices the market will be in favour of the seller while if the ask prices is below the MEMM prices the opposite is the case. This also gives us an indication of whom takes the greatest risk in the market.

4.1.1. *Microsoft.* The calibration data for the Microsoft stock is until September 18, 2006 so for comparison with the calculated MEMM prices we take bid/ask prices from September 18, 2006, for a range of options with different strikes and maturities. The spot price at the time was \$26.85 and we assume a fixed interest rate of 4.94%, which was the three month treasury yield at the time. The Microsoft stock is highly traded and liquid, and the option market for Calls and Puts has good liquidity as well.

Looking at the illustration in Figure 7(a) we see that MEMM prices are significantly lower than the bid prices for Call options, which is also true for Put options. This clearly suggests that the market is in favour of the issuer of the claim, letting the buyer take on the largest part of the risk. Hence, the market prices are such that the seller gets a compensation for bearing the risk being short the options. Of course, the buyer knows the maximal loss when entering the position, whereas the seller needs to take into account that the position needs to be liquidized or hedged in order to control potential and uncertain losses. We notice that the difference increases with time to exercise, reflecting that the future is more uncertain than the present, leading to a higher risk premium. In this respect, we can not disregard the possibility that the market operates with a higher or lower interest rate than the 4.94% we used in the simulations. However this should have opposite effect on Call and Put prices, making one of them even more in favour of the issuer. Looking at the implied Black-Scholes volatilities, Figure 7(b), we see that the volatilities, as the prices suggest, are close for short maturities, displaying a skewed smile. For long maturities the implied volatility for the MEMM prices is close to zero, now with more of a smirk than a smile. The bid prices are almost constant for the Call options while Put display a flat smile.

It appears that the market prices is not consistent with the prices under the MEMM. As we see from Figure 7(c) the difference between the bid prices and the MEMM prices are peaking around the spot price with a 10-12% mispricing. One may speculate that this may be a reflection of the instability of hedging portfolio around the strike, where one can have big changes in the hedging position when the spot is close to the strike. The hedge is more stable when the strike is farther from spot (in either direction), and thus a hedge does not need to be updated so frequently to be accurate. Looking at the percentage error in Figure 7(d) gives another perspective, showing that the mispricing of the options with very small price is substantially higher. The MEMM prices of far-out-of-the-money options are counted in fractions of hundreds or thousands of a dollar. Quoted prices of these options, on the other hand, are usually around five or ten cents, giving a percentage error close to 100%. One should bear in mind that the volume traded at the quoted prices is insignificant, if not zero, for the mentioned options.

To conclude, we have that the prices under MEMM are not close to the quoted prices but significantly lower than both bid and ask prices. This tells us that for the Microsoft option the risk in the market is carried by the buyer of the options. The large observed difference indicates that the market perceives a higher risk aversion than zero, which is assumed in the MEMM prices. In the next section we will investigate the risk aversion further.

4.1.2. *Volvo.* The Stockholm stock exchange is substantially smaller compared to the stock markets in the United States. Compared to NYSE average daily dollar volume of 56.1 billions in 2005 the Stockholm stock exchange's 14,876 million Swedish crowns are rather trifling. Together with the late introduction of options on stocks listed on Stockholm stock exchange makes it a much less liquid market. We expect the Volvo options to be traded less frequently than the Microsoft options, which is indicated by volume data. We are interested in if there is any obvious differences in the risk aversion due to this fact, or if the same features as for Microsoft is visible for the Volvo options. The Volvo options are quoted on December 30, 2005 with a stock price at the time of 374.5 SEK. We assumed a interest rate of 3%, which was close but slightly higher than the 3-month STIBOR at the time, but there was a general consensus at the time that the Swedish central bank would increase the repo rate during the year.

The main difference compared to Microsoft is that the MEMM prices for Volvo is above the bid prices for a large range of strikes and maturities for Call options, more precisely, far-in-the-money options. This is illustrated through the implied volatility in Figure 8. Thus, the buyer's prices may be decisive for the trades. The bid prices for in-to-the-money options result in a indistinguishable small implied volatility, which means that the bid price is close to the present value of the payoff from the option. For out-of-the-money options, the implied volatilities are above the MEMM price with a implied volatility around 15-20%. The ask prices is above the MEMM prices for all Call options and looking at Put options there are only a few cases of bid prices falling below the MEMM prices. As observed above the price difference peak around the strike but the percentage error is not as grave as for the Microsoft options. This due to the nominal value of the stocks, higher nominal price

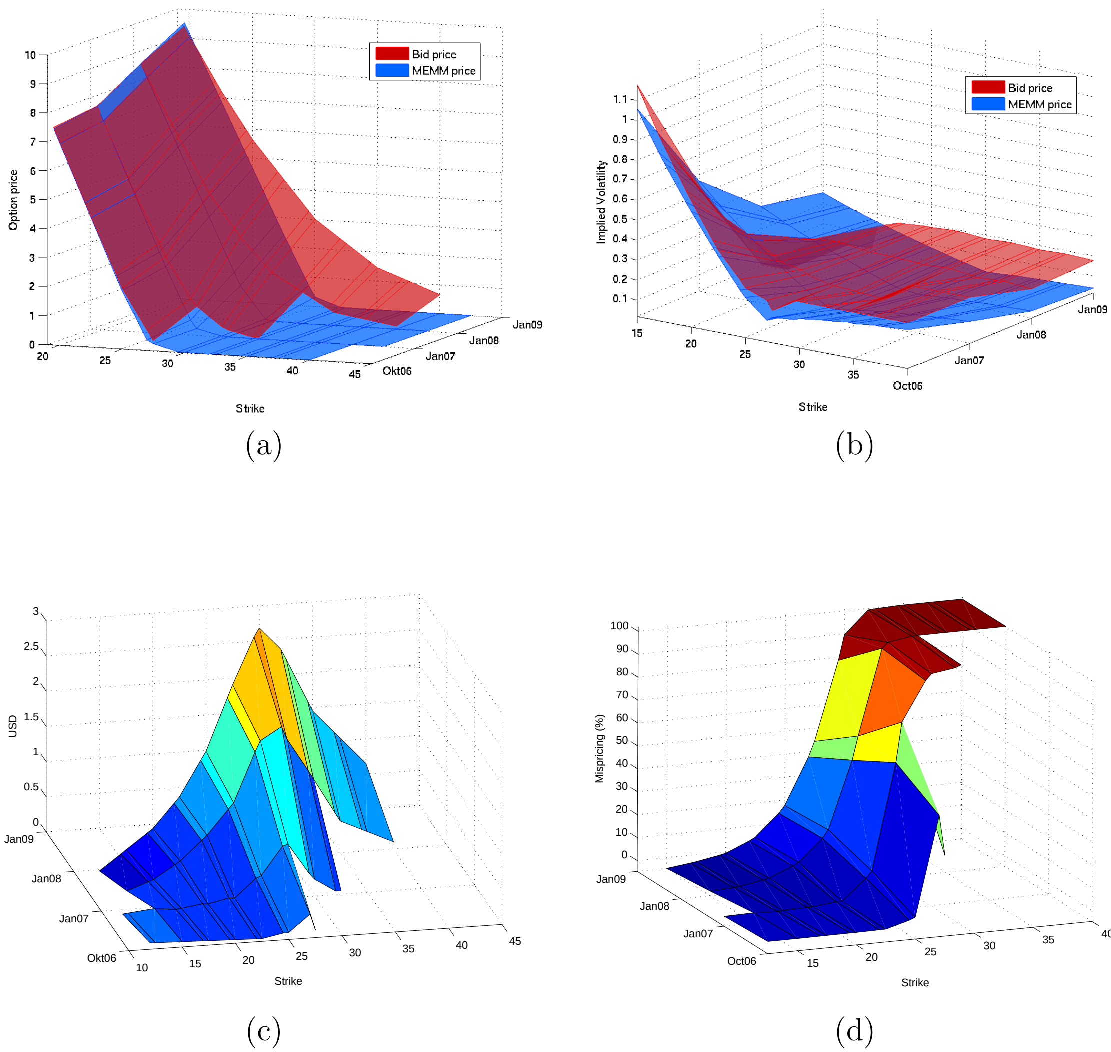
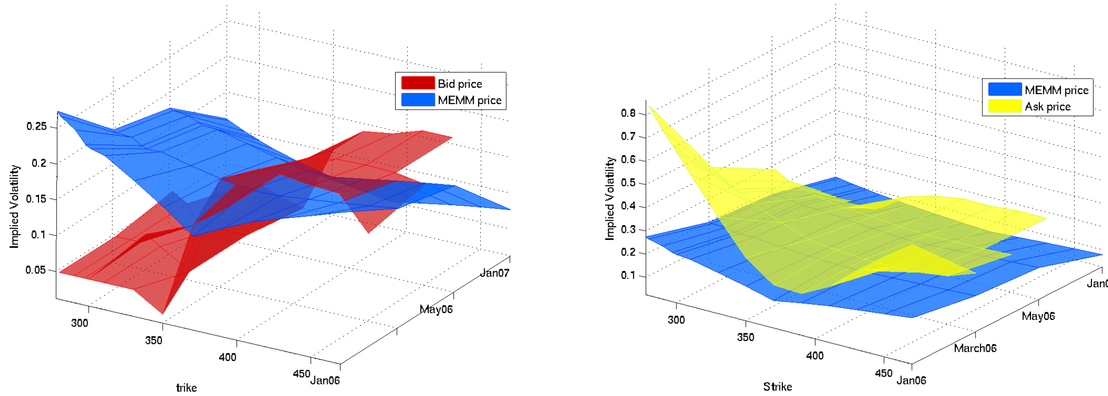


FIGURE 7. Illustrations of features and differences between theoretical MEMM prices and bid prices for Microsoft call options taken September 18, 2006 . *(a):* Option prices. *(b):* Implied volatility. *(c):* Difference between MEMM prices and market prices. *(d):* Mispricing in percentage error between MEMM prices and bid prices.

of the stock gives higher nominal value for options on the flanks. This in turns makes the percentage error appearing to be less severe.



(a) (b)

FIGURE 8. *(a):* Plot of implied Black-Scholes volatilities of call options on the Volvo stock, bid prices from December 30, 2005 and simulated MEMM prices.*(b):* Plot of implied Black-Scholes volatilities of call options on the Volvo stock, ask prices from December 30, 2005 and simulated MEMM prices.

5. The implied risk aversion

In this section we calculate the implied risk aversion γ from quoted (bid/ask) Call and Put option prices. We proceed as follows. For a given option price, we iterate γ until we reach an agreement between the market quote and the indifference price. For each iteration of γ, we use the numerical algorithm to solve the integro-partial differential equation as described in detail in the previous section. For the root-fining we use Ridder's method as described in Press et al. [20], avoiding taking a numerical derivative. In general the algorithm executes in 5-7 iterations but in some cases the double is needed.

We have collected the results for Microsoft in two figures (Figs. 9-10), where the implied risk aversion as a function of maturity and strike of Put and Call options are plotted, respectively.

The implied risk aversion for Put options is decreasing with the strike price. There is the apparent effect that the risk aversion decreases more sharply to the left of to current spot price (compounded by the interest rate up to exercise time) than to the right. In fact, for some maturities we even see an increase with the strike to the right. The opposite effect is observed for Call options in Fig. 10, where the implied risk aversion is increasing with the strike, but more sharply to the right of today's spot price (compounded by the interest rate up to exercise time). This tells us that for Put options the market is averse to crashes, meaning that the Put option becomes far-in-the-money. The same effect is for Call options, where big price jumps upward brings the options far-in-the-money. We can reflect back this relative high risk aversion towards such abrupt and big price changes to the underlying asset price model, which seems to not capture the sudden big movements in prices as much as desired by the market. We know that the model produces a volatility smile, but despite this and the modelling of the heavy-tailed logreturns, the market still is pricing in the fear of a crash (or the opposite for Call options). One may mend this (at least for Put options), by introducing a leverage effect in the stock price model, however, for mathematical and numerical complexity we have not done so (see, however, Rheinländer and Steiger [21]).

From the option prices we notice that Calls with high strike close to maturity have unrealistic high prices, taking in to consideration they are unlikely to be exercised. Clearly the price is there to make a market and not as a fair price. For the options far from maturity we see a slight upward slop opposite to what we observed for the Put options. However, for the options with an exercise date in the near future we see that the aversion is higher for low strikes and falling towards the spot price. This could be a consequence of the amounts of money involved in transactions with Call options with low strikes. The unboundedness of the payoff functions could make this a very costly deal in terms of transactions and money transfers. It could be that the issuers marks up the price to cover expenses inflicted upon them for this. This could also explain why this feature is not as prominent for the Put options, since the payoff is bounded in that case.

The aversion towards market crashes is also signatured in the decreasing implied risk aversion with time to maturity for far-out-the-money Put options. For Put options being close to maturity, we face a market crash risk, while the longer to maturity, the less is the reason for such options to be striked due to a market crash. Hence, we clearly see the effect of crash risk in the risk aversion, which is not clear at all in the price difference between bid and MEMM (see Fig. 7(c)), however clear in the implied volatility (see Fig. 7(b)). The opposite picture holds for Call options, naturally, since here it is the possibility for an upward jump that worries the issuer, and which is difficult to hedge on the short term.

Another effect is that the risk aversion flattens with increasing time to maturity. When the exercise time is far in the future, the market seems to have a more overall view on the risk, with an aversion being less dependent on the strike. This is in line with the understanding that the sample space for the asset prices are more spread in the future, and we have weaker information on whether the option will be striked or not. In the long term, large price movements will have time to even out, thus controlling the issuer's risk of being striked. However, the overall picture of an de/increasing risk aversion holds for Put/Call options, as for the case when we have short time to maturity.

As we can see from Figs. 11- 12, the same conclusions hold for the implied risk aversion from Volvo.* Surprisingly, the implied risk aversions for both Call and Put options seem to be approximately one order *lower* than for Microsoft. In view of liquidity, one may have expected the opposite effect. However, on the other hand, the implied risk aversion is a complicated nonlinear function of many effects, including the model parameters like the distribution and volatility. Thus, it is not clear how the liquidity comes in and affects the risk aversion for our situation.

*Note that we have only considered the implied risk aversion from those prices which are bigger than the MEMM prices, that is, being the issuer's prices

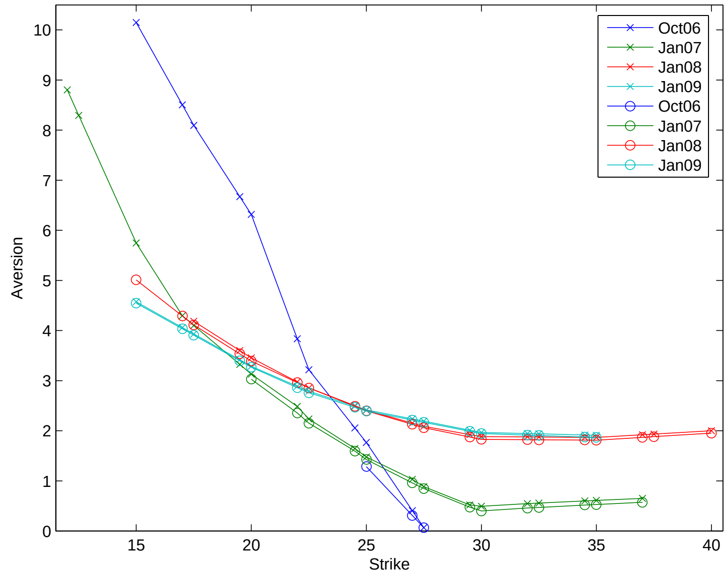


FIGURE 9. Plot of market aversion simulated from Microsoft put options, bid/ask prices from September 18, 2006.

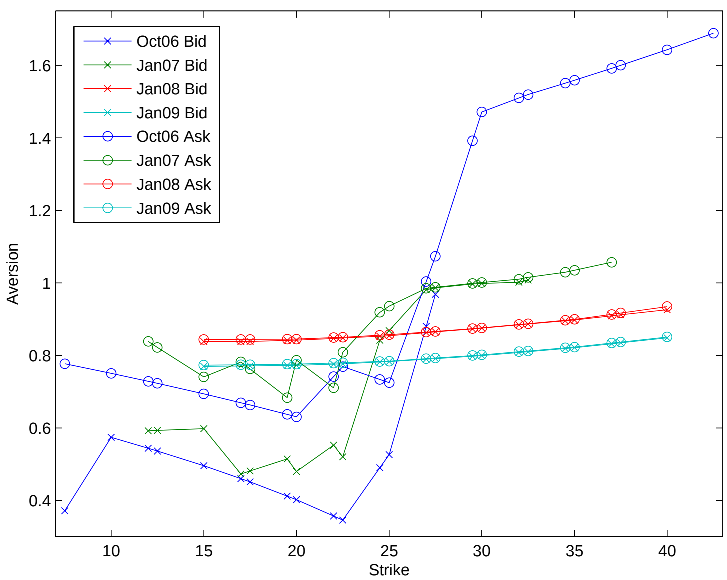


FIGURE 10. Plot of market aversion simulated from Microsoft call options, bid/ask prices from September 18, 2006.

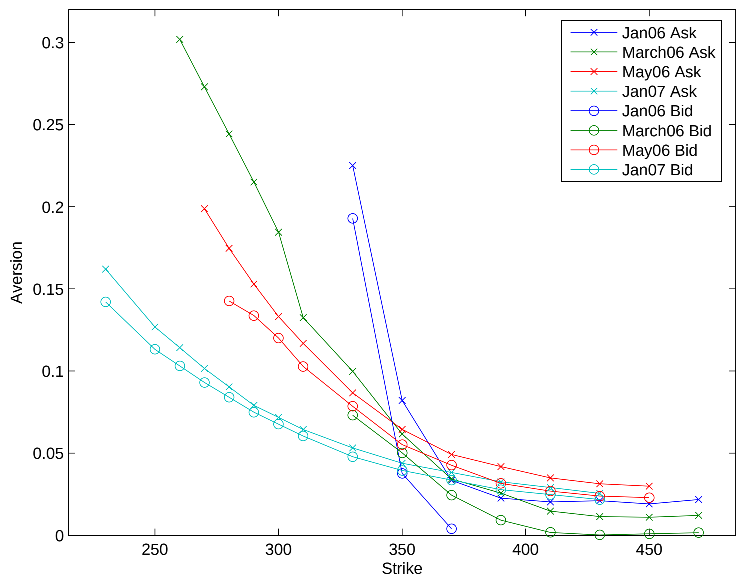


FIGURE 11. Plot of market aversion simulated from Volvo put options, bid/ask prices from December 30, 2005.

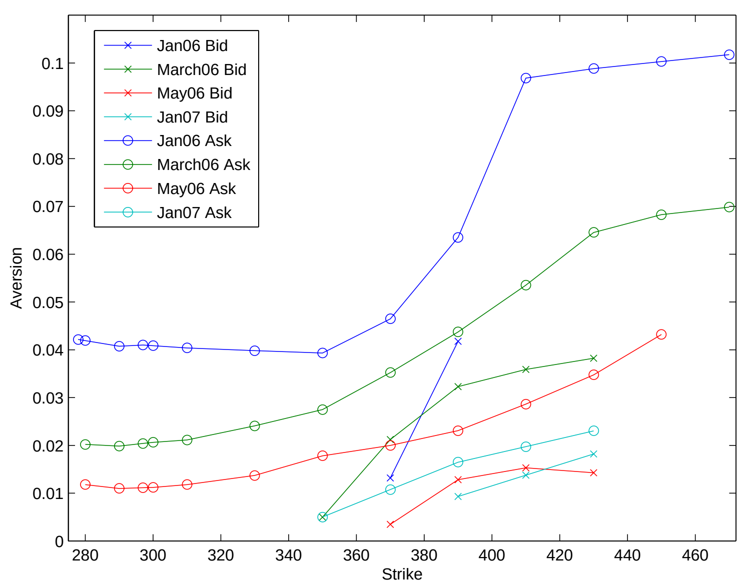


FIGURE 12. Plot of market aversion simulated from Volvo call options, bid/ask prices from December 30, 2005.

Microsoft Put Options

Maturity	Strike	MEMM price	Bid price	Ask price
October 06	15.0	0.00	.	0.05
	17.5	0.00	.	0.05
	20.0	0.00	.	0.05
	22.5	0.00	.	0.05
	25.0	0.03	0.05	0.10
	27.5	0.76	0.75	0.75
	30.0	3.00	3.00	3.00
January 07	12.0	0.00	.	0.05
	15.0	0.00	.	0.05
	17.0	0.00	.	0.05
	19.5	0.00	.	0.10
	20.0	0.00	0.05	0.10
	22.0	0.00	0.10	0.20
	22.5	0.00	0.10	0.15
	24.5	0.00	0.30	0.35
	25.0	0.01	0.35	0.40
	27.0	0.18	0.95	1.05
	27.5	0.40	1.20	1.25
	29.5	2.15	2.60	2.65
	30.0	2.62	3.30	3.10
	32.0	4.57	5.50	5.10
	32.5	5.06	5.50	5.60
	37.0	9.47	10.10	10.10
January 08	15.0	0.00	0.05	0.20
	17.5	0.00	0.15	0.25
	20.0	0.00	0.35	0.50
	22.5	0.00	0.75	0.75
	25.0	0.00	1.25	1.30
	27.5	0.03	2.15	2.25
	30.0	1.22	3.50	3.70
	35.0	5.86	7.90	8.10
	40.0	10.53	12.90	13.10
January 09	15.0	0.00	0.20	0.25
	20.0	0.00	0.70	0.75
	22.5	0.00	1.15	1.25
	25.0	0.00	1.85	1.95
	30.0	0.36	4.10	4.20
	35.0	4.28	7.90	8.10

TABLE 2. Prices for puts on the Microsoft stock. Bid and Ask prices from the September 18, 2006, MEMM prices simulated with parameter estimates from Section 3.1.

Microsoft Call Options

Maturity	Strike	MEMM price	Bid price	Ask price
October 06	7.50	19.39	19.40	19.60
	10.00	16.90	17.00	17.10
	12.50	14.41	14.50	14.60
	15.00	11.92	12.00	12.10
	17.50	9.44	9.50	9.60
	20.00	6.95	7.00	7.10
	22.50	4.46	4.50	4.70
	25.00	2.00	2.10	2.20
	27.50	0.24	0.35	0.40
	30.00	0.01	.	0.05
	32.50	0.00	.	0.05
January 07	12.00	15.06	15.00	15.20
	15.00	12.12	12.10	12.20
	17.00	10.15	10.10	10.30
	19.50	7.70	7.70	7.80
	20.00	7.20	7.20	7.40
	22.00	5.24	5.30	5.40
	22.50	4.75	4.80	5.00
	24.50	2.79	3.10	3.20
	25.00	2.31	2.65	2.75
	27.00	0.51	1.25	1.35
	27.50	0.23	1.00	1.05
	29.50	0.02	0.35	0.40
	30.00	0.01	0.25	0.30
	32.00	0.00	0.05	0.10
	32.50	0.00	0.05	0.10
January 08	15.00	12.83	12.50	12.70
	17.50	10.49	10.30	10.50
	20.00	8.16	8.20	8.30
	22.50	5.82	6.20	6.30
	25.00	3.48	4.30	4.50
	27.50	1.18	2.85	2.95
	30.00	0.03	1.70	1.75
	35.00	0.00	0.45	0.50
	40.00	0.00	0.10	0.20
January 09	15.00	13.51	12.90	13.10
	20.00	9.06	9.00	9.20
	22.50	6.84	7.20	7.40
	25.00	4.62	5.60	5.70
	30.00	0.32	3.00	3.20
	35.00	0.00	1.45	1.60
	40.00	0.00	0.65	0.75

TABLE 3. Prices for calls on the Microsoft stock. Bid/Ask prices from Sept 18, 2006, MEMM prices simulated with parameters estimated in Section 3.1.

Volvo Put Options

Maturity	Strike	MEMM price	Bid price	Ask price
January 06	330.00	0.03	0.15	0.55
	350.00	0.33	0.40	0.60
	370.00	3.02	3.65	4.50
	390.00	16.13	15.50	18.00
	410.00	34.85	33.00	37.00
	430.00	54.64	53.00	57.00
	450.00	74.56	73.00	76.75
	470.00	94.49	93.00	97.00
March 06	280.00	0.00	.	1.00
	290.00	0.02	.	1.00
	300.00	0.03	.	1.00
	310.00	0.11	0.03	0.50
	330.00	0.59	1.20	2.20
	350.00	2.44	4.50	5.50
	370.00	7.76	10.50	12.00
	390.00	18.82	20.75	24.75
	410.00	34.67	35.25	39.25
	430.00	52.98	53.25	57.25
	450.00	72.29	73.00	76.75
	470.00	91.92	93.00	97.00
May 06	270.00	0.01	0.25	1.00
	280.00	0.03	0.70	1.25
	290.00	0.08	1.40	1.70
	300.00	0.14	2.20	2.40
	310.00	0.33	5.75	3.65
	330.00	1.13	11.00	7.25
	350.00	3.33	20.50	13.50
	370.00	8.47	32.25	23.00
	390.00	18.88	48.00	37.00
	410.00	34.01	65.50	52.25
	430.00	51.57	84.50	69.75
	450.00	70.30	.	88.50
January 07	230.00	0.00	0.40	1.35
	250.00	0.00	1.00	2.10
	270.00	0.00	2.55	3.90
	290.00	0.01	5.00	6.00
	310.00	0.03	8.75	10.25
	330.00	0.15	13.50	16.25
	350.00	0.67	20.75	23.75
	390.00	9.44	41.25	46.00
	430.00	42.55	70.50	74.75

TABLE 4. Prices for puts on the Volvo stock. Bid/Ask prices from December 30, 2005, MEMM prices simulated with parameters estimated in Section 3.2.

Volvo Call Options

Maturity	Strike	MEMM price	Bid price	Ask price
January 06	277.88	97.11	95.50	99.50
	290.00	85.01	83.25	87.25
	297.04	77.98	76.25	80.25
	310.00	65.04	63.25	67.25
	330.00	45.10	43.25	47.25
	350.00	25.45	23.75	27.50
	370.00	8.18	8.75	10.50
	390.00	1.35	2.00	2.40
	410.00	0.15	.	1.00
	430.00	0.01	.	1.00
	450.00	0.00	.	1.00
	470.00	0.00	.	1.00
March 06	280.00	96.30	94.50	98.50
	290.00	86.37	84.50	88.50
	300.00	76.45	74.75	78.75
	310.00	66.59	65.00	69.00
	330.00	47.21	46.25	50.25
	350.00	29.21	29.25	32.75
	370.00	14.70	16.50	19.00
	390.00	5.97	8.00	9.50
	410.00	2.06	2.95	4.25
	430.00	0.62	0.90	1.90
	450.00	0.17	0.02	1.00
May 06	280.00	97.79	95.00	99.00
	290.00	87.95	85.00	89.00
	300.00	78.13	75.25	79.25
	310.00	68.44	65.75	69.75
	330.00	49.49	47.50	51.50
	350.00	31.95	31.25	35.50
	370.00	17.39	19.00	21.75
	390.00	8.13	10.50	12.00
	410.00	3.62	5.00	6.25
	430.00	1.54	2.05	3.05
	450.00	0.64	0.60	1.55
January 07	290.00	93.37	85.50	89.50
	310.00	74.01	67.75	71.75
	330.00	54.74	52.00	56.25
	350.00	35.88	39.00	43.25
	390.00	6.10	20.75	23.25
	430.00	0.61	10.00	11.50

Table 5. Prices for calls on the Volvo stock. Bid/Ask prices from December 30, 2005, MEMM prices simulated with parameters estimated in Section 3.2.

References

[1] Andersen, L., Andreasen, J., (2000): Jump-diffusion models: Volatility smile fitting and numerical methods for pricing, *Review of derivatives research*, 4, 231-262.

[2] Barndorff-Nielsen, O. E., Shephard, N. (2001): Modelling by Lévy processes for financial econometric, in: O. E. Barndorff-Nielsen, T. Mikosch and S. Resnick (Eds.), *Lévy Processes - Theory and Applications*, Boston: Birkhäuser, 283-318.

[3] Barndorff-Nielsen, O. E., Shephard, N. (2001): Non-Gaussian Ornstein-Uhlenbeck-based models and some of their uses in financial economics, *Journal of the Royal Statistical Society*: Series B 63 (with discussion), 167-241.

[4] Becherer, D. (2006): Bounded solutions to backward SDE's with jumps for utility optimization and indifference hedging. To appear in *Annals of Applied Probability*.

[5] Benth, F. E., Karlsen, K. H., Reikvam, K. (2003): Merton's portfolio optimization problem in a Black and Scholes market with non-Gaussian stochastic volatility of Ornstein-Uhlenbeck type, *Mathematical Finance* 13(2), 215-244.

[6] Benth, F.E., Groth, M. (2005): The minimal entropy martingale measure and numerical option pricing for the Barndorff-Nilsen - Shephard stochastic volatility model, *Submitted*.

[7] Benth, F. E., Meyer-Brandis, T. (2005): The density process of the minimal entropy martingale measure in a stochastic volatility model with jumps, *Finance and Stochastics* 9, 563-575.

[8] Black, M., and Scholes, M. (1973): The Pricing of Options and Corporate Liabilities. *Journal of Political Economy* 81(3), 637-654.

[9] Cont, R., Voltchkova, E., (2003): Finite difference methods for option pricing in jump-diffusion and exponential Lévy models, *Finance and Stochastics*, 9, 299-325.

[10] Engle, R. F., Ng, V. K., Rothschild, M. (1990): Asset pricing with a factor-ARCH covariance structure, *Journal of Econometrics* 45, 213-237.

[11] S. Gudonov (1959). Finite difference methods for numerical computations of discontinuous solutions of the equations of fluid dynamics. *Matematiceskij Sbornik*, 47, pp. 271–306.

[12] Hilber. N., Matache, A., Schwab, C. (2005): Sparse Wavelet Methods for Option Pricing under Stochastic Volatility, *Journal of Computational Finance*, 8(4).

[13] Hodges, S. D., Neuberger, A. (1989): Optimal replication of contingent claims under transaction costs, *Review of Futures Markets* 8, 222-239.

[14] Lindberg, C. (2006): News-generated dependence and optimal portfolios for n stocks in a market of Barndorff-Nielsen and Shephard type, *Mathematical Finance* 16(3), 549-568.

[15] Lindberg, C. (2006): Portfolio optimization and a factor model in a stochastic volatility market, *Stochastics* 78(5), 259-279.

[16] Lindberg, C (2006): The estimation of a stochastic volatility model based in the number of trades, *submitted*.

[17] Matache, A., von Petersdorff, T., Schwab, C. (2004): Fast deterministic pricing of options on Lévy driven assets. *in Mathematical Modelling and Numerical Analysis* 38(1), 37-72.

[18] Merton, R. (1973): Theory of Rational Option Pricing. *Bell Journal of Economics and Management Science* 4(1), 141-183.

[19] Nicolato, E., Venardos, E. (2003): Option pricing in stochastic volatility models of the Ornstein-Uhlenbeck type, *Mathematical Finance* 13, 445-466.

[20] Press, W.H., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P. (1992): *Numerical Recipes in C*, Cambridge: Cambridge University Press.
[21] Rheinländer, T., and Steiger, G. (2006). The minimal entropy martingale measure for general Barndorff-Nielsen/Shephard models, *Annals of Applied Probability*, 16(3), pp. 1319–1351.
[22] G. Strang (1968). On the construction and comparison of difference schemes. *SIAM J. Num. Anal.*, 5, pp. 506–517.

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